Laboratory 2

In-Plane Beam Bending with the VIC-EDU Measurement System

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I. OVERVIEW

The purpose of this laboratory is to utilize optical imaging systems with stereo digital image correlation within VIC-EDU to make full-field, non-contacting, surface deformation measurements with the educational measurement system for a simply supported beam specimen subjected to centrally located loading. As part of this laboratory, students will be provided basic theoretical background to predict the surface displacements and strains that they will measure. To obtain the measurements, students will learn how to (a) prepare the specimen surface and apply a usable, high-contrast speckle pattern, (b) arrange the VIC-EDU system to acquire images that can be used for stereo analysis, (c) calibrate the VIC-EDU system for stereo DIC, (d) acquire and store images during mechanical loading of the specimen, (e) perform post-processing to obtain results using the stereo images and (f) compare the experimental results with the theoretical solutions.

> **NOTE** Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".

II. BEAM THEORY

A specimen for which the well-known Euler-Bernoulli beam theory is applicable is shown schematically in Fig. 1a, where the theoretical formulation generally requires that $L >> h >> t$. Typical ratos for $\frac{h}{L}$ and $\frac{t}{h}$ are on the order of 0.5, 0.33, 0.25 or even lower, depending upon experimental limitations for the application. If these conditions are met, and the deformations are small for the centroidal coordinate system shown in Fig. 1b, we can write the following expression for the vertical displacement of the beam.

$$(1) \qquad \frac{d^2v}{dx_c^2} EI_{z_c z_c} = M_{z_c}\left(x_c\right)$$

where E is the elastic modulus (units of Pa or psi) of the material; $I_{z_c z_c}$ is the area moment of inertia of the beam cross-section about the centroidal z_c axis; v is the vertical displacement of the centroidal $z_c - x_c$ surface at distance x_c along the beam; $M_{z_c}(x_c)$ is assumed to be a positive internal reaction moment as a function of the distance, x_c, along the beam; $F_{y_c(x_c)}$ is the vertical shear force on the cross-section; C is the centroid of the beam cross-section. The specimen provided with the VIC-EDU system has an estimated modulus of elasticity, $E \sim 6$ GPa. As described in a NOTE in Section VI of this lab, there may be variability in E due to manufacturing differences for the as-supplied specimen material.

For the rectangular beam shown in Fig. 1b, the location of the centroid is $(x_c, \frac{h}{2}, \frac{t}{2})$ relative to the bottom left corner of the beam (see Fig. 2). The origin for the centroidal coordinate system can be chosen anywhere along x_c for the beam specimen. However, in practice, the origin is oftentimes chosen at the left end of the beam specimen so that x_c is positive along the entire length of the beam.

In addition to the vertical displacement, $v(x_c)$ in Eq. 1, when cross-sections shown in Fig. 2 remain planar during mechanical loading, then there is a normal stress, $\sigma_{x_c x_c}$, on each cross-section that is given as;

$$(2) \qquad \sigma_{x_c x_c} = \frac{-M_{z_c}\left(x_c\right)y_c}{I_{z_c z_c}}$$

where y_c is vertical distance from the neutral surface. The stress distribution corresponding to Eq. 2 is shown in Fig. 3.

If the beam is assumed to be loaded only on the top and bottom surfaces in the vertical directions and the beam is an isotropic, homogeneous, linear elastic material, then the corresponding strains along the x_c direction can be written, Eq. 3.

$$(3) \quad \varepsilon_{x_c x_c} = \frac{-M_{z_c}(x_c) y_c}{EI_{z_c z_c}}$$

Thus, horizontal lines above the centroid ($y_c > 0$) on the front surface will be compressed and horizontal lines below the centroid ($y_c < 0$) will be stretched. Furthermore, according the Eq.(3), the strain $\varepsilon_{x_c x_c}$ varies linearly with vertical distance from bottom to top of the beam in the same way as the stress distribution varies in Fig. 3. In this laboratory, the external load will be applied at or very near the center of the beam, $x_c \approx \frac{L}{2}$.
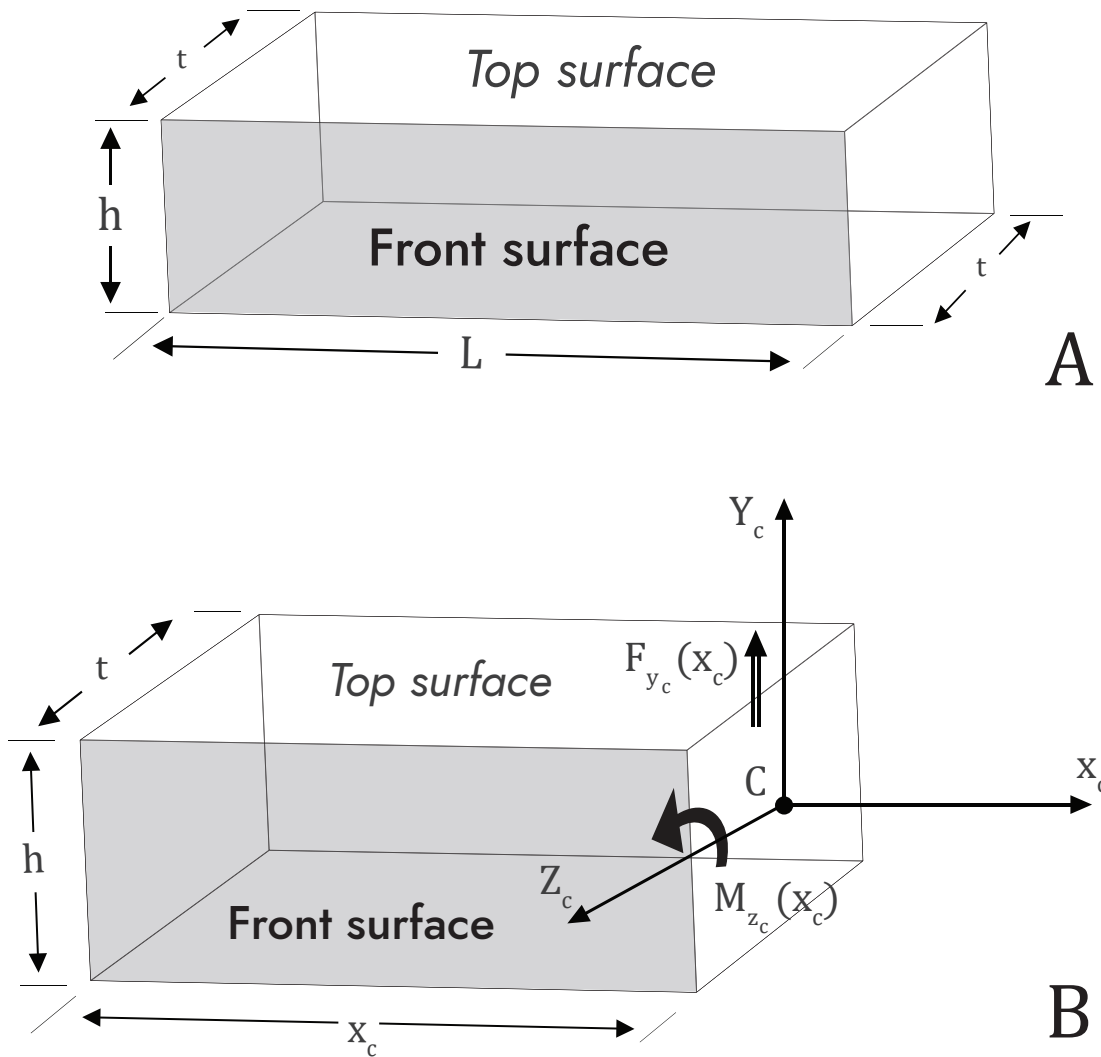

Figure 1: Beam specimen schematic with coordinate system and positive directions for internal static reactions

II.a. Implications of Theory for Measurements on Front Surface of Beam

As shown in Eq. 1, points along the centerline of the beam will move vertically, either down (negative movement) or up (positive movement) when a mechanical load is applied and a bending moment is developed within the beam. Thus, you should expect that the front surface will move vertically and you will measure how the vertical displacement, $v(x_c)$, changes as you move horizontally across the beam.

In addition to vertical movement of lines on the front surface, Eq. 3 indicates that points on the front surface will be stretched on one side of the centroid and compressed on the other side of the centroid. Thus, if there is sufficient spatial resolution in the measurements across the height of the beam, there should be a variation in the measured strain $\varepsilon_{x_c x_c}$ that is approximately linear in the y_c direction as you move up and down on the front surface of the beam.
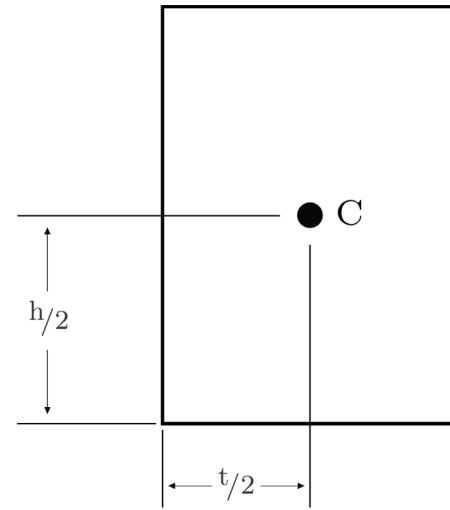

Figure 2: Rectangular cross-section and centroid location

Another important implication of the theory is that you can PREDICT or ESTIMATE both the vertical displacements and axial strains that will occur in your specimen prior to loading. If the elastic modulus of the beam, dimensions of the beam, end conditions (how beam is supported at ends) and the range of mechanical loads to be applied are known, then static equilibrium can be used to obtain the internal moment and Eq. 1 can be solved for the vertical displacement along the beam. The expression for $v(x_c)$ that is obtained can be used to determine whether the experimental displacements should be large enough to be measured by the VIC-EDU system for some of the loads in the range that is to be applied. You do this by comparing your predictions to the as-supplied lower bound (+/-0.02mm for in-plane displacement). This is extremely useful in practice as you will know a priori whether you are well above the lower bounds and the experiment will be productive or not.

III. EXPERIMENTAL PREPARATIONS

III.a. Specimen Preparation

To make accurate DIC measurements using images of a specimen, the specimen must be prepared and the FRONT SURFACE patterned with high contrast speckles having the appropriate size and an approximately random distribution across the field of view. The procedures to do this are detailed in Lab 1 and in Application Note AN-1701 [3] and are briefly summarized here:

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, the surface is initially painted white
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE
- After the white paint has dried, apply a random black dot pattern with size of 0.026" (0.635 mm) using the supplied roller and ink pad. Patterning should require approximately 2-3 passes in order to achieve the high contrast 50/50 ratio of white to black that is best. Do not try to align the roller passes, as attempts to move the roller around during patterning will smear or blur the dots, potentially degrading the quality of stereo DIC measurements. When done, image the front surface and check the pattern with VIC-3D (EDU) tools.
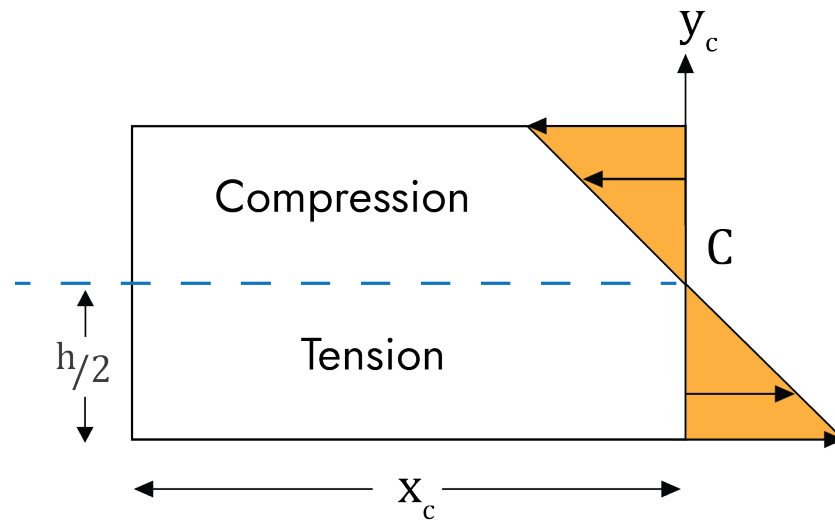
Figure 3: Schematic of axial stress distribution on vertical cross-section of beam

III.b. Installing Specimen in Loading System

Once the specimen has been prepared, the specimen will be placed in a mechanical loading frame so that the boundary supports for the beam are known with reasonable approximation. For example, if the beam is placed on two "knife edges" or rollers, then these points could be considered to be SIMPLE SUPPORTS for static analysis of the beam. Vertical loading in y_c direction will be applied on the top surface, preferably by a loading fixture that applies load along a Z_c-oriented line that spans the thickness of the beam.

III.c. VIC-EDU System Preparation

Once the specimen is in the loading frame, the tripod should be located in front of the specimen and the VIC-EDU system firmly attached to the tripod mounting head. At this point, the VIC-EDU system should be powered up and oriented to obtain images of the front surface of the specimen. Once the system is operational, the procedures outlined in Lab 1 and the User's Manual for focusing the specimen and preparing to acquire images should be used to locate the system in a position where focused images can be obtained. At this point, calibration of the system should be performed.

III.d. VIC-EDU System Calibration

Once the VIC-EDU system is mounted to the tripod and placed in position where the specimen is in sharp focus, system calibration is performed. Though in principle the entire system could be picked up and moved carefully to a new location for calibration, and then moved back into position for viewing the specimen without affecting the measurements, such movements may introduce changes in the optical imaging system that are not readily apparent and can affect the accuracy of the results. To minimize the potential for such errors, it is recommended that you calibrate in front of the test specimen. There should be enough depth of field to allow for this. Lighting provided within the VIC-EDU measurement head should provide bright, clear images of the as-supplied calibration target.

When performing calibration, it is recommended that the target occupy 80-90% of the field of view. For this experiment, the supplied target dot pattern has black dots with a 14mm spacing on a white background. Furthermore, it is required that all three of the black dots containing a central white dot be within the field of view of both cameras. These dots provide the necessary information for estimating rotation of the planar target.

To perform calibration, the target should be in focus for both cameras. Since the focus is fixed on the VIC-EDU system, the target should be located approximately 0.50m in front of the system. Then, the target is rotated several times and focused images acquired. The rotations should include motions about three separate orthogonal axes to ensure accurate identification of all calibration parameters. Typically, between 25 and 50 calibration images are acquired. Please refer to Lab 1 and the User's Manual for additional information regarding calibration and processing of the calibration images.
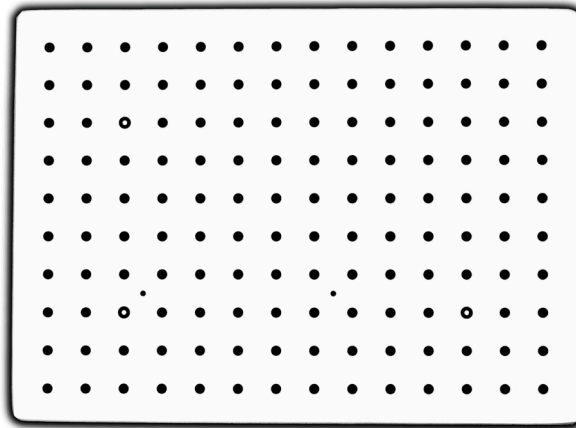

Figure 4: Photograph of a typical calibration target used to calibrate stereo-vision measurement systems such as VIC-EDU

IV. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is successfully completed in front of the specimen, the target is removed so the front surface of the beam specimen already placed in the loading frame is visible and in focus.

- Read Lab 1 and VIC-EDU User's Manual regarding the procedure for initiating VIC-3D (EDU) to acquire and store image pairs for this experiment.
- Acquire several image pairs of the speckled front surface of the specimen in the unloaded state.
- Apply the first increment of load/displacement and acquire two to three pairs of images, recording the image numbers and the corresponding applied load/displacement.
- Continue load/displacement application and image acquisition process until you reach the pre-identified maximum load or displacement for this specimen.
- Terminate the image acquisition process and the experiment via the process described in Lab 1, e.g., close VIC-Snap (EDU).
- Unload the specimen.

At this point, the experimental process is complete and the deformation data is embedded in the image pairs that were acquired during the experiment. To extract full-field deformation data, the VIC-3D (EDU) software is initiated to analyze the images. Please refer to the detailed description in Lab 1 for how to initiate VIC-3D (EDU) and extract displacement and strain data.

- Step size selection: The pixel distance in each direction between subsets that are analyzed within the ROI. See pgs 8-10 in User's Manual.
- Initial Guess: In most cases, the software does not need an initial guess. However, if the motions are large due to the loading, then the VIC-3D (EDU) software may require an initial guess before beginning the analysis. The process for doing this is described on pgs 9-11 in the User's Manual.

With this information, the VIC-EDU image analysis software can be initiated using the *Start Analysis* icon. This will open a window that gives additional analysis options, as discussed on pgs 10-13 in the User's Manual. Please refer to the discussion provided in the User's Manual to make your selection. Once all analysis options have been selected, the analysis is initiated via the Run button. After analysis has been completed, the measurement data is now available for post-processing.

Post-Processing and Plots
A rather complete discussion of various output variables and how to display the results is given on pages 14-19 in the User's Manual. You should review this in detail prior to initiating detailed processing of the data.

V. POTENTIAL SOURCES OF DIFFERENCES BETWEEN PREDICTIONS & MEASUREMENTS

For the vertical displacement, $v(x_c)$, the analytical results using Eq. 1 require you to assume specific boundary conditions at $x_c = 0$ and $x_c = L$. If you assumed that the $v(0) = 0$, but the support actually moved downwards during loading, then this will affect the comparison. Since this is also true at the right end of the beam, the entire $v(x_c)$ curve may have to be shifted up or down so that the ends match the assumed boundary conditions. Such motions are RIGID BODY MOTIONS (RBMs) and are quite common in experiments. The presence of RBMs may need to be accounted for when looking at the comparison of experimental and theoretical displacements. Additional issues that could affect the quality of the comparison between theory and experiment include:

- Slight shifts in the load position relative to the assumed location ($x_c = \frac{L}{2}$)
- Accuracy of the measured geometric lengths
- Accuracy of the assumed modulus of elasticity
- Slight temperature changes
- Potentially other factors that are not noted here (e.g. vibrations).

Interestingly, since the strains obtained from VIC-EDU are defined in a way that eliminates the effect of RBM, if a VIC-EDU system has sufficient spatial resolution across the height, the bending strain field measurements would be accurate and independent of RBM.

An additional source of potential errors is the way in which the experimental data is processed. For example, the experimental strains obtained from VIC-EDU use the measured displacements in a region, with a "finite-size strain filter" to estimate the strains at a point. The strain filter has various sizes over which it acquires the strain. This process may introduce slight shifts in the strain relative to the actual strain at the corresponding point on the specimen.

VI. LABORATORY RESULTS TO BE PRESENTED

1. Measure the specimen geometry ($L, h, $ t) several times with a micrometer or similar device and provide a table of the individual results, mean value ($\overline{X}$) for each dimension and standard deviation, S_x for each dimension. The basic equations are given below;

$$\overline{X} = \sum_{i=1}^{n} \frac{x_i}{n} \quad ..Mean\ Value$$

$$S_x = \left[\sum_{i=1}^{n} \frac{(x_i - \overline{X})^2}{n-1} \right]^{\frac{1}{2}} \quad Standard\ Deviation\ (S.D.)$$

Note: The range on the modulus of elasticity, E, for the test material is 3 GPa ≤ E ≤ 9 Gpa, with the expected value being 6 Gpa.

2. Assuming beam boundary conditions that are consistent with the actual experiment just performed, solve Eq. 1 to obtain the predicted displacement of the centerline of the beam, $v(x_c)$. Show your analysis and final equation for $v(x_c)$. As a hint, your equations should give $v\left(\frac{L}{2}\right) = -\frac{PL^3}{48EI_{z_c z_c}}$ if $v(0) = v(L) = 0$.

   2.a. Verify that your $v(x_c)$ equation satisfies your assumed boundary conditions at left side and right side of the beam. Show the equations with these values inserted to demonstrate results.

3. Using the equation for $v(x_c)$ in Question 2 with the measurements obtained in Question 1, obtain equations for the mean value and standard deviation in the predicted vertical displacement at the center of the beam, $x_c = \frac{L}{2}$. Do not consider variations in load or load position, only variations in the geometric variables measured in Question 1.

4. Using the VIC-EDU system, acquire two pairs of test images of the as-patterned specimen after it has been centered, focused and properly exposed, but without any load. Both pairs of images should be taken without moving the specimen. Include one pair of test images for the 3 point bend specimen in the report.

   - The two image pairs are to be analyzed using the VIC-EDU software to obtain and report data over a central region of the specimen. As you are setting up the analysis, fill in the table below (same format as in Lab 1) to provide a list of all options/parameters used in the analysis. Note that this table should be filled out for all stereo DIC experiments, without exception, to have a record of the analysis parameters being used.
   - Plot the initial shape of the specimen, $Z(x, y)$, to see whether it is nominally flat or not.

5. Estimate the location of the beam centerline on your image and then extract the vertical displacement data, V_i, using VIC-EDU at "i" discrete locations along the length of the beam from $0 \le x_c \le L$ for six applied loads.

$$P = 0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5} \ and \ P_{max}..$$

   Here, we assume that $x_c = 0$ corresponds to the centerline of the beam above the left support and $x_c = L$ corresponds to the centerline of the beam above the right support.

   5.a. Determine the specimen shape in the unloaded state.

   5.b. For each of the applied loads, specimen dimensions and elastic modulus of the beam, determine whether the predicted, theoretical displacement at each location x_i is above the lower bound limit of the VIC-EDU measurement system.

6. Plot the predictions and experimental data, $v(x_i)$ and $V(x_i)$, at each x_i location where experimental data was obtained for each of the six applied loads on the same graph and discuss the results.

7. Select vertical lines that lie along the y_c direction at $x_c = 0.40L$ and $x_c = 0.60L$. For these lines, use Eq. 3 to predict the strain ε_{xx} at several y_j locations from $-\frac{h}{2} < y_c < \frac{h}{2}$ for $P = P_{max}$. Note that you must use your static analysis to obtain moment $M_{z_c}(x_c)$ at these x_c-locations.

   7.a. Discuss whether the strain distribution you calculated is measurable with the VIC-EDU system or not.

8. Summarize results for displacements and for analytic strain predictions.

Lab 2 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N pixels^2)	
Strain filter type	
Auto-plane fit	Yes or No

NOTES

VII. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.